JOURNAL OF AIRCRAFT
Vol. 47, No. 1, January–February 2010

Effect of Uncertainty on Hub Vibration Response of Composite Helicopter Rotor Blades

Young Hyun You* and Sung Nam Jung†
Konkuk University, Seoul 143-701, Republic of Korea
Prashant M. Pawar‡
Shri Vithal Education and Research Institute, Tal-Pandharpur 413 304, India
and
Eui Sup Shin§
Chonbuk National University, Jeonju 561-756, Republic of Korea

DOI: 10.2514/1.44477

In this study, an assessment is made to quantify the influence of random material properties and fabrication/manufacturing uncertainties on the aeroelastic response and hub vibratory loads of composite rotor blades. The random variables include lamina stiffness properties, ply thicknesses and fiber orientation angles of the laminate structures, and the elastic-axis offset from the aerodynamic center in the section of the blade. The stochastic behavior of the random variables obtained from previous experimental or analytical studies are used to evaluate the stochastic behavior of the cross-sectional stiffnesses of the blades. The uncertainties in the stiffness properties result in the dissimilarity of the rotor system, which brings extra non-N_b/rev vibrations. It is observed that the probability histograms of N_b/rev hub vibratory loads exhibit a normal distribution, whereas those of non-N_b/rev hub loads show non-Gaussian-type distributions. Numerical results showing the effects of material and geometric uncertainties on the aeroelastic response and hub vibration behavior of composite rotor blades are illustrated and important conclusions are drawn based on the observations.

Nomenclature

E_1, E_2	=	Young's moduli
EI_y	=	flap bending stiffness
EI_z	=	lag bending stiffness
e_d	=	aerodynamic center offset from the elastic axis
F_X, F_Y, F_Z	=	hub forces (longitudinal, lateral, vertical)
GJ	=	torsion rigidity
G_{12}	=	shear modulus
M_X, M_Y, M_Z	=	hub moments (roll, pitch, yaw)
M_y, M_z	=	blade bending moments (flap, lag)
m_0	=	reference mass per unit length
N_b	=	number of blades
R	=	length of the blade
U	=	axial displacement
β_y, β_z	=	cross-sectional rotations
γ	=	Lock number
θ	=	ply orientation angle
ν_{12}	=	Poisson's ratio
ϕ	=	elastic twist angle
ψ	=	azimuth angle
Ω	=	rotational speed
$\dot{}$	=	time derivative, $\partial/\partial t$

Subscript

x	=	spatial derivative, $\partial/\partial x$

Superscript

T	=	transpose of a vector

Received 19 March 2009; revision received 28 August 2009; accepted for publication 9 September 2009.

*Graduate Research Assistant, Department of Aerospace Information Engineering.
†Professor, Department of Aerospace Information Engineering. Senior Member AIAA.
‡Professor and R&D Cell Coordinator, College of Engineering, Department of Mechanical Engineering, Pandharpur, Gopalpur.
§Associate Professor, Department of Aerospace Engineering.

I. Introduction

FIBER-REINFORCED composite materials are widely used in the aircraft industries because of their superior fatigue characteristics, greater damage tolerances, and larger stiffness-to-weight ratios as compared with their metal counterparts [1–4]. In helicopter rotor applications, composite materials can bring additional features like drastic reduction in the number of parts and bulkiness, especially for the rotor hub system, which is typically a large source of the profile drag. Despite these advantages, composite materials generally increase the level of uncertainties for the overall structural system. The uncertainties range from the statistical nature of the material properties of constituent ingredients (e.g., fibers and resin) to the randomness in the fabrication (e.g., layup and curing) and manufacturing processes. The uncertainty could affect the global behavior of the composite rotor system through the individual action of the blades.

Assessment of uncertainty in the prediction of aeroelastic response of flight vehicles is a complex process that involves various nonlinearities and interactions between structural and aerodynamic disciplines. A review paper by Pettit [5] gives a comprehensive survey on the effects of uncertainties in the aeroelastic analysis, design, and testing of fixed-wing aircraft. The sources of uncertainties and the quantification of their influence on the aeroelastic response of aircraft structures are also investigated. A great deal of research on uncertainty quantification has been reported in the literature; however, most of the uncertainty studies have focused on the fixed-wing aircraft [6–8].

The rotorcraft uncertainty analysis appears more complicated than that of the fixed-wing aircraft, due to the unsymmetrical nature of the lift pattern around the rotor disk and the coexistence of nonrotating and rotating components that play key roles in generating thrust and control moments of the vehicle. Murugan et al. [9] investigated the effects of uncertainties of composite materials on the cross-sectional stiffness properties, natural frequencies, and aeroelastic responses of composite helicopter rotor blades. Stochastic behaviors of composite material properties available in the form of experimental data along with Monte Carlo simulation technique were used to evaluate the statistical analysis. They have demonstrated that uncertainty in the composite material properties got propagated into aeroelastic response, which led to large deviations, particularly in the higher harmonic components of rotating blade loads. The uncertainty analysis of blade responses was further extended to include fixed-frame hub vibration analysis [10]. It was found that the 4/rev vibratory loads exhibit considerable deviations from the baseline deterministic values due to the material uncertainty. They have considered identical uncertainties in all the four blades that affect only on 4/rev vibratory loads. However, it will be more realistic to consider independent uncertainties in each of the blades, which essentially brings a blade-to-blade dissimilarity.

Typically, the main rotor system is the principal source of vibration in the helicopter. The sources of vibrations can be divided into two types: 1) vibrations due to the asymmetric nature of a rotor in forward flight, which are present even in case of balanced rotor (tracked rotor) system, and 2) vibrations due to the blade-to-blade dissimilarity, which results from manufacturing uncertainties, environmental effects, and highly vibratory loads [11]. The latter will also be caused by uncertainties in the composite material properties and the fabrication processes of the blades. Several researchers [12–15] focused on predicting the rotor vibrations by assuming a dissimilarity in the rotor system induced by various faults for the structural health monitoring or the vibration analysis. Roget and Chopra [16] demonstrated the trailing-edge-flap approach for the reduction of hub loads induced by dissimilar rotor systems. Recently, Pawar and Jung [17] demonstrated use of the active twist control method for the vibration reduction of composite rotor blades with a dissimilar rotor system.

The present study aims at quantifying the influence of uncertainties encountered in the fabrication and manufacturing stages of composite helicopter blades on the blade responses and hub loads. Various sources of uncertainties, including the material and geometric variables, are investigated in the aeroelastic analysis. Special focus is given to introducing a dissimilarity in the rotor system arising from the stochastic properties of the composite blades that may bring new harmonics of vibrations. The mechanical material properties of composite laminas E_1, E_2, G_{12}, and ν_{12}; layer thicknesses t_i; fiber orientation angles θ_i of constituent laminas; and the elastic-axis offset e_d from the aerodynamic center for the cross section of the composite blades are treated as random variables and these are used to determine the stochastic behaviors of the cross-sectional stiffness values, aeroelastic response, and hub vibration behavior. A comprehensive aeroelastic analysis system suitable for composite helicopter rotor blades with dissimilarities is employed for the vibratory load analysis of the rotor system. The stochastic behaviors of hub vibratory loads are determined using the Monte Carlo simulation coupled with the aeroelastic analysis system.

II. Aeroelastic Analysis of Dissimilar Rotor System

For the comprehensive aeroelastic analysis of composite rotor blades, the cross-sectional properties of the blade should be determined a priori. The mixed beam approach developed by Jung et al. [18] is used to obtain the stiffness constants that represent extension, torsion, flap bending, lag bending, and couplings. This theory can model thin-walled multicelled beams with open and closed cross sections. The influence of elastic couplings, shell wall thickness, transverse shear, warping, and warping restraint effects is taken into account in the beam formulation. The 5×5 stiffness matrix $\bar{\mathbf{K}}$ relating the generalized beam force vector $\bar{\mathbf{F}}$ and the generalized displacement vector $\bar{\mathbf{q}}$ can be obtained following the procedures described in [19] as

$$\bar{\mathbf{F}} = \bar{\mathbf{K}}\,\bar{\mathbf{q}} \tag{1}$$

where

$$\bar{\mathbf{F}} = \lfloor N \quad T_s \quad M_y \quad M_z \quad M_\omega \rfloor^T \tag{1a}$$

$$\bar{\mathbf{q}} = \lfloor U_{,x} \quad \phi_{,x} \quad \beta_{y,x} \quad \beta_{z,x} \quad \phi_{,xx} \rfloor^T \tag{1b}$$

where N is the axial force; M_y and M_z are flap bending moment and lag bending moment, respectively; T_s is the St. Venant twisting moment; and M_ω is the Vlasov bimoment. In addition, U, β_y, β_z, and ϕ are the axial displacement, cross-sectional rotations about the y and z axes, and elastic twist deformation, respectively. The $\bar{\mathbf{K}}$ in Eq. (1) represents the beam stiffness matrix at an Euler–Bernoulli level of approximation for the extension and bending and Vlasov level for the torsion [18].

Uncertainties in the cross-sectional stiffness properties of a composite rotor blade inevitably introduce dissimilarities in the rotor system. A comprehensive aeroelastic analysis system based on the finite element method in both space and time [20] is used to evaluate the blade response and hub vibration response of a helicopter with a dissimilar rotor system. For the aeroelastic analysis, the helicopter is represented by a nonlinear model of rotating elastic blades dynamically coupled to a six-degree-of-freedom rigid fuselage. Each blade undergoes flap (out-of-plane) bending, lag (in-plane) bending, elastic torsion, and axial displacements. The governing equations of motion for the blades are derived using Hamilton's principle applicable to a nonconservative system:

$$\delta\Pi = \int_{\psi_1}^{\psi_2} (\delta U - \delta T - \delta W_e)\,\mathrm{d}\psi = 0 \tag{2}$$

where ψ is the azimuth angle measured along the rotor disk; δU, δT and δW_e are the virtual variation of strain energy, the variation of kinetic energy, and the external virtual work done, respectively; and $\delta\Pi$ represents the total potential of the system. It is noted that δU and δT include energy contributions from components that are attached to the blades (e.g., pitch link and lag damper). The aerodynamic forces acting on the blades contribute to the virtual work variational δW_e. The aerodynamic forces and moments are calculated using the unsteady aerodynamic model developed by Leishman and Beddoes [21] along with a free-wake model developed by Bagai and Leishman [22].

Applying the finite element method into the discretized form of Hamilton's principle, one can construct the nonlinear equations of motion for blades expressed in terms of global nodal degrees of freedom $\boldsymbol{q}$, which can be written in symbolic form as

$$\mathbf{M}\,\ddot{\mathbf{q}}(\psi) + \mathbf{C}(\mathbf{q},\psi)\dot{\mathbf{q}}(\psi) + \mathbf{K}(\mathbf{q},\psi)\mathbf{q}(\psi) = \mathbf{F}(\psi,\mathbf{q},\dot{\mathbf{q}}) \tag{3}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{F}$ are the global inertia, damping, stiffness matrix, and load vector, respectively. The blade structure is discretized into a number of beam finite elements. A 15-degree-of-freedom beam element composed of two end nodes and three internal nodes is used to describe the flap–lag–torsion behavior of hingeless composite blades [23]. In addition, the blade response along the rotor azimuth is obtained by using the temporal finite element technique, in which the time period of one rotor revolution is discretized into a series of time finite elements. The periodic boundary condition is imposed by connecting the first and last time finite elements.

For an isotropic rotor system, only the vibratory load harmonics that are multiples of the number of blades (designated as mN_b/rev, where m is the integer and N_b is the number of blades) are transmitted to the fuselage through the rotor hub. For a dissimilar rotor system, however, non-N_b/rev harmonics would also appear and transmit to the fixed-frame hub parts. Unlike the isotropic blade analysis, when blades are dissimilar with each other, the blade response should be evaluated individually [16,17]. Steady and vibratory components of blade loads for the mth blade are calculated using the force summation method. In this method, blade inertia and aerodynamic forces are integrated directly over the length of the blade for blade loads. The fixed-frame hub loads are then obtained by summing the contributions from individual blades. A coupled trim procedure is used to obtain the blade responses, pilot control inputs, and vehicle orientations in a simultaneous manner. The detailed formulation for the derivation of equations and their discretization procedures are given in [20,23].

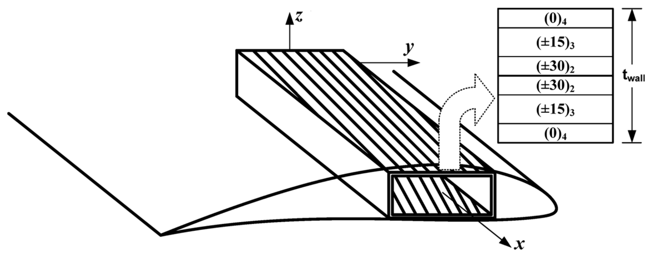


Fig. 1 Schematic of the blade and its layup.

III. Results and Discussions

The effects of stochastic variation of material properties and fabrication/manufacturing uncertainties on the cross-sectional stiffness values, aeroelastic responses, and hub loads for four-bladed composite hingeless rotor blades are investigated using the Monte Carlo simulation (MCS). The uncertainties introduced herein are classified into three parts: 1) material uncertainty in the composite laminas, 2) fabrication uncertainty with regard to ply thicknesses and fiber angles of composite laminates, and 3) manufacturing uncertainty on the elastic-axis offsets from the aerodynamic center of the blade. The baseline (deterministic) blade is modeled as a single-cell thin-walled composite box beam that matches with the realistic magnitudes of cross-sectional stiffness, inertia, and rotating frequencies of a stiff in-plane helicopter blade [24]. Figure 1 shows the schematic of the composite blade and its layup geometries for the wall of the section. The respective dimensions of the box section are outer width of 203.2 mm, outer depth of 38.1 mm, and wall thickness of 3.556 mm. The mechanical properties are of AS4/3501-6 graphite/epoxy lamina. Each wall of the box section consists of a balanced layup as $[0_4/(15/-15)_3/(30/-30)_2]_s$. The flight conditions of the model helicopter are advance ratio $\mu = 0.3$, thrust level $C_T/\sigma = 0.07$, and lock number $\gamma = 6.34$. In addition, tail rotor radius $r_{tr}/R = 0.2$, solidity $\sigma_{tr} = 0.15$, and tail rotor location and offset from the main rotor hub are $x_{tr}/R = 1.2$ and $h_{tr}/R = 0.2$. Other vehicle properties are flat-plate area $f/\pi R^2 = 0.01$ and hub vertical offset $h/R = 0.2$. A shaft-fixed condition along with the propulsive trim iteration is used for the present aeroelastic analysis.

A. Stochastic Behavior of Cross-Sectional Stiffnesses

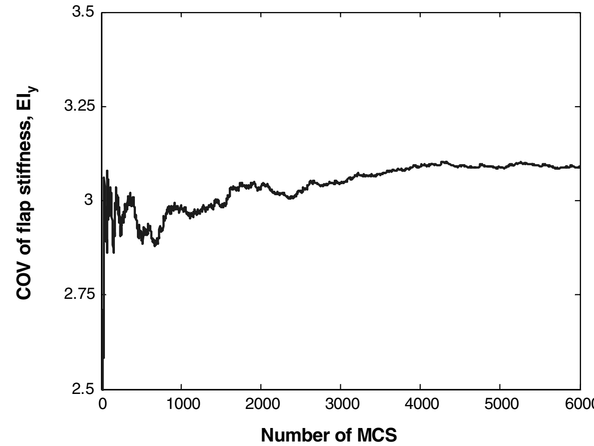
The random material properties and fabrication uncertainties are considered to first evaluate the stiffness constants of the composite rotor blade. Earlier studies [25–28] have demonstrated that the randomness of data is present in the composite material laminas (material uncertainty) and also their laminate structures (fabrication uncertainty). They found that the normal (Gaussian) distribution with a deviation could be a reasonable fit to characterize most of the random variables. Table 1 shows the mean standard deviation (SD) and statistical distribution of the random variables used in the material and fabrication uncertainty. To make the analysis more realistic, all the values given in Table 1 are taken from experimental results [25,26,28].

For the estimation of stochastic behavior of composite blades by the material and fabrication uncertainty, it is important to decide the number of samples required for the analysis. The number of samples can be decided based on the convergence of coefficients of variation (COV). The COV is defined as the standard deviation divided by the mean value. Figure 2 shows the COV results for the flap bending stiffness EI_y, lag bending stiffness EI_z, and torsion rigidity GJ with respect to the MCS evaluations. As is seen, at least 4000 to 5000 evaluations are required to obtain the sufficiently convergent solution. In the present study, 6000 randomly chosen samples are evaluated unless otherwise stated. To obtain the MCS results, it generally takes about 2.5 h for the blade stiffnesses and about 60 h for the aeroelastic responses and hub load calculation. The computation is performed on a 3 GHz dual-core PC with 4 GB main memory.

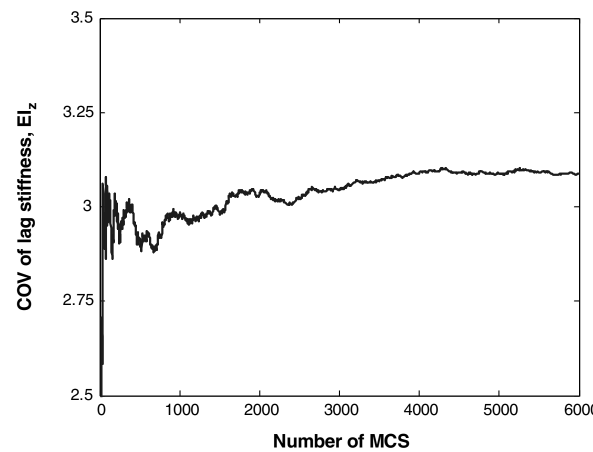
Figure 3 shows the histograms for each of the cross-sectional stiffness properties, taking into account the material uncertainty of composite blades denoted with percentage deviations from the mean values. It is seen that the random material properties of composite

Table 1 Stochastic properties of random variables

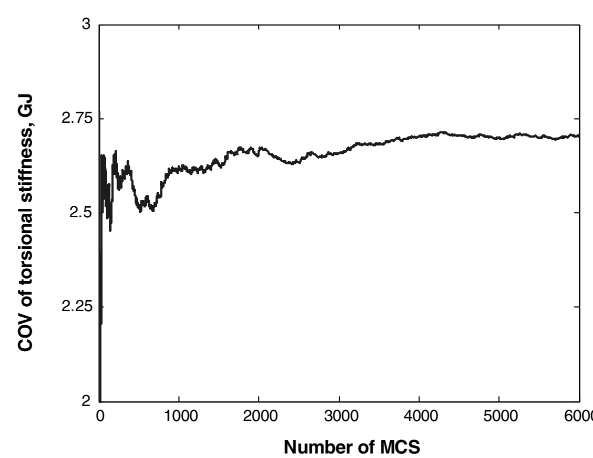
Properties	Mean value	Standard deviation	Distribution
E_1, GPa	141.96	4.812	Normal
E_2, GPa	9.79	0.418	Normal
G_{12}, GPa	6.00	0.256	Normal
ν_{12}	0.42	0.0153	Normal
ply thickness, mm	5	0.25	Normal
Ply angle, deg	0	1.8	Normal



a) Flap bending stiffness

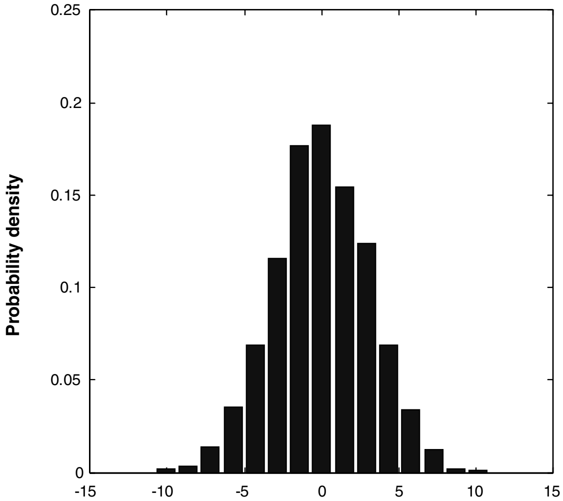


b) Lag bending stiffness

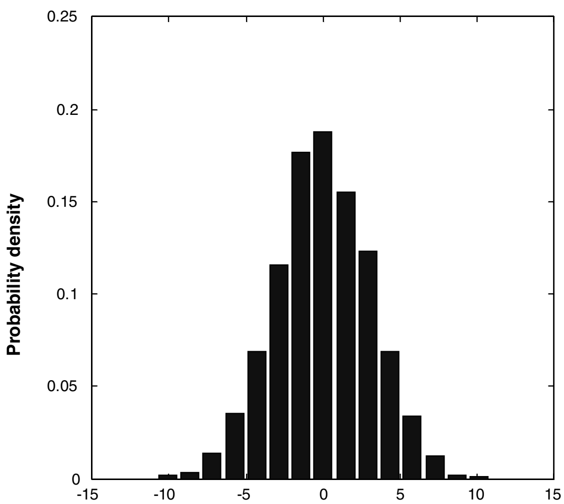


c) Torsion stiffness

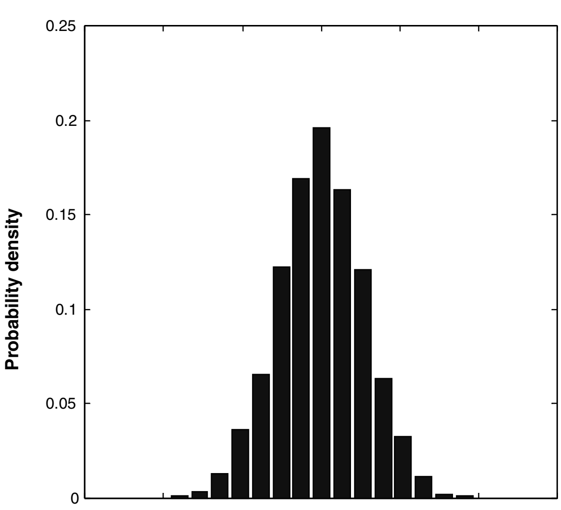
Fig. 2 COV of the beam stiffnesses using MCS for the cross-sectional stiffnesses.



a) % deviation of EI_y



b) % deviation of EI_z



c) % deviation of GJ

Fig. 3 Probability distribution of cross-sectional stiffnesses of the blade with the material uncertainty.

laminas become transmitted to the cross-sectional stiffnesses, resulting in a normal distribution. Table 2 shows the mean and COV values for the cross-sectional stiffnesses when either the material or fabrication uncertainty is considered. The material uncertainty case shows about two-times-larger variation in bending stiffnesses and similar order of magnitudes in the torsion stiffness, as compared with the fabrication uncertainty case. In summary, the material uncertainty affects more on the blade stiffness values than with the fabrication uncertainty. Similar observations have been found in Shaker et al. [26], in which stochastic analysis for the free vibration behavior of laminated composite plates was performed.

B. Stochastic Behavior of Aeroelastic Response

The random characteristics of the cross-sectional stiffness properties also affect the aeroelastic response of the helicopter rotor blades. Figure 4 shows the variations in tip responses due to the stochastic behavior of flap, lag, and torsion stiffness properties of the composite blades. In this case, only the material uncertainty leading to about 3% changes of COV in the material properties is considered (see Table 2). Uncertainties in the cross-sectional stiffnesses are seen to have a substantial influence on the blade tip responses. The peak-to-peak response variations of flap, lag, and torsion deflections are about 15.2, 20.3, and 10.2%, respectively, in comparison with the baseline (deterministic) values denoted as solid black lines in Fig. 4. Since larger stiffness values of blades generally induce smaller structural responses, the lowest responses arise from the combination of the highest values of beam stiffnesses and vice versa. As can be seen from Fig. 3, either the highest or the lowest stiffness value has additional $\pm 10\%$ to the baseline values. The aeroelastic response of the blade with the fabrication uncertainty shows similar patterns with smaller variations in the magnitudes and is thus omitted due to the limited space.

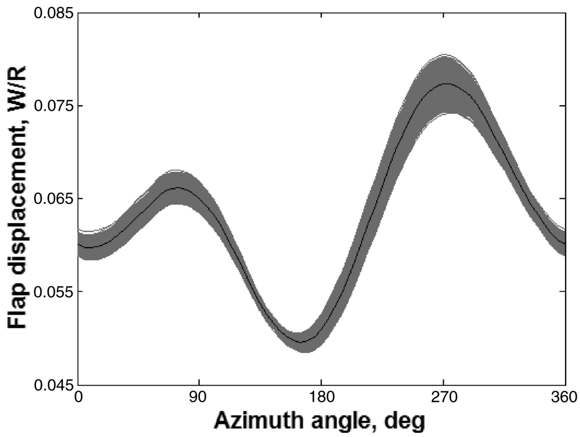
The variation of shear centers among the production blades is often encountered during the manufacturing process, and thus the elastic-axis (shear center) offsets from the aerodynamic center are treated as random variables. The Monte Carlo simulation results with this manufacturing uncertainty are presented in Fig. 5, in which the tip responses of the blade are presented as a function of azimuth angles. A SD of 1.5% chord length is assumed for the randomness of elastic axis along the chord line of the blade. For simplicity of the analysis, the mean of elastic-axis offsets is set to zero. It is assumed that the locations of center of gravity and elastic axis coincide with each other. It is defined as positive when the elastic axis is located toward the leading edge from the aerodynamic center (quarter-chord).

As can be seen in Fig. 5, a large variation of tip responses, especially for the flap and torsion deformation, is obtained with the manufacturing uncertainty introduced through the randomness of elastic-axis offsets, whereas the chordwise (lag) displacements show only small variations. Since there is no offset between the center of gravity and the elastic axis, the dramatic changes in flap and torsion responses have no relation with the aeroelastic instability (e.g., flutter), but with the increase of aerodynamic loads as the offsets become greater. It is noted that the maximum variations of elastic-axis offsets are within about $\pm 5\%$, because the SD values is assumed as 1.5% at a 95% probability. Figure 6 shows the changes of flap bending and torsion moments (nondimensionalized each by $m_0 \Omega^2 R^3$) at the blade root obtained for the two extreme values ($e_d = \pm 5\%$) as compared with the baseline (zero-offset) value. As is expected, the overall behavior of the blade loads (Fig. 6) has quite a similar waveform with their resulting responses (Fig. 5). The peak-to-peak magnitudes of both the flap bending moments and flap displacements, especially with -5% offset, become significantly changed with respect to the baseline zero-offset case. The reason for

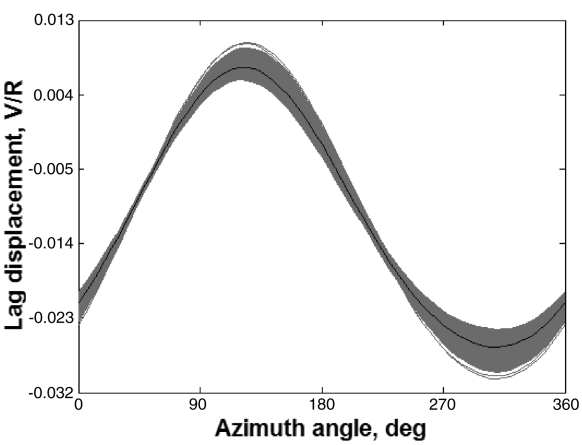
Table 2 Statistics of cross-sectional stiffness values

Stiffness	Mean, N/m^2	COV, % Material uncertainty	COV, % Fabrication uncertainty	Distribution
EI_y	47.8e3	3.053	1.428	Normal
EI_z	761e3	3.052	1.410	Normal
GJ	22.8e3	2.678	1.962	Normal

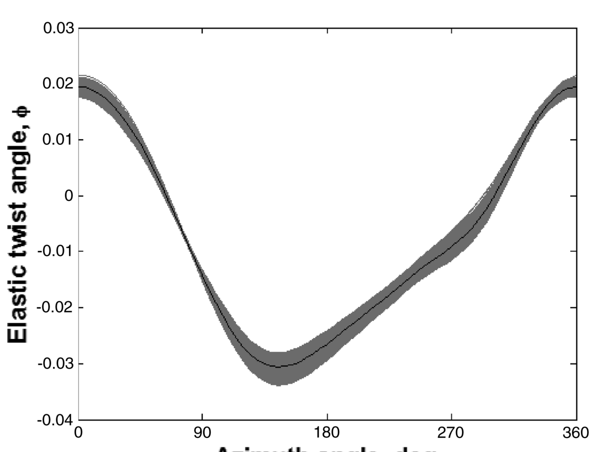
this is due to that fact that the negative elastic-axis offset induces a pitch-up, leading to increased aerodynamic loads. The other extreme case (+5% offset) shows much reduced loadings and responses compared with the −5% offset case but still exhibits substantial deviations from the baseline results. Based on this observation, it can be argued that the random characteristic encountered in the manufacturing process of composite rotor blades can increase the prediction errors in a manner that the conventional deterministic approach may lead to unrealistic solutions, especially for the flap and torsion responses of the blade.



a) Flap displacement



b) Lag displacement



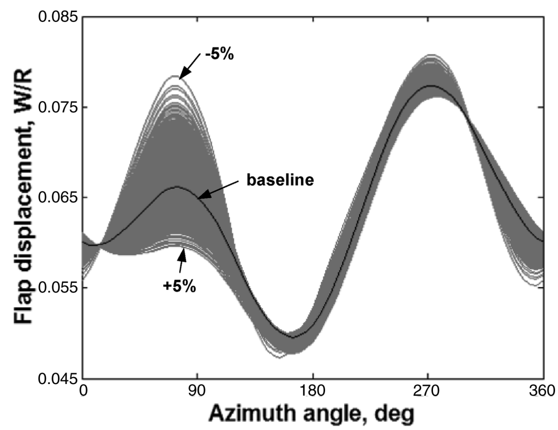
c) Elastic twist angles

Fig. 4 Stochastic distribution of tip responses of composite blades with the material uncertainty.

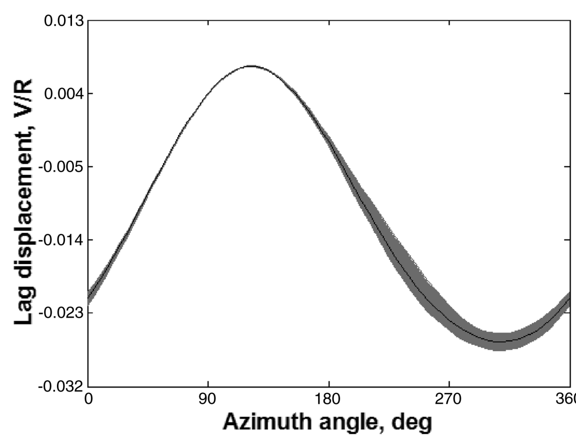
C. Stochastic Behavior of Hub Vibratory Loads

Next, the stochastic behavior of hub vibratory loads is investigated using the statistical data obtained for the cross-sectional stiffness values. When the randomness of the stiffness properties is considered, the cross-sectional properties of all the participating blades in the rotor system will not be identical with each other, which inevitably brings dissimilarities to the rotor system. In this study, 6000 different sets of the rotor system with dissimilarity are used in the comprehensive aeroelastic analysis to obtain the stochastic behavior of the vibratory loads transferred through the rotor hub.

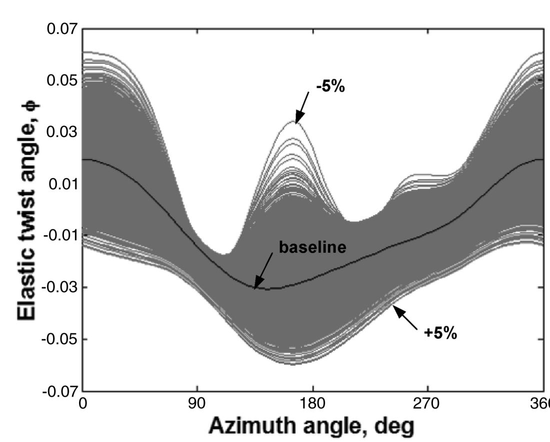
Figure 7 shows the mean values of 1/rev to 5/rev of all the hub forces and moments: F_X (longitudinal), F_Y (lateral), F_Z (vertical), M_X (roll), M_Y (pitch), and M_Z (yaw) for the 6000 samples considered



a) Flap displacement

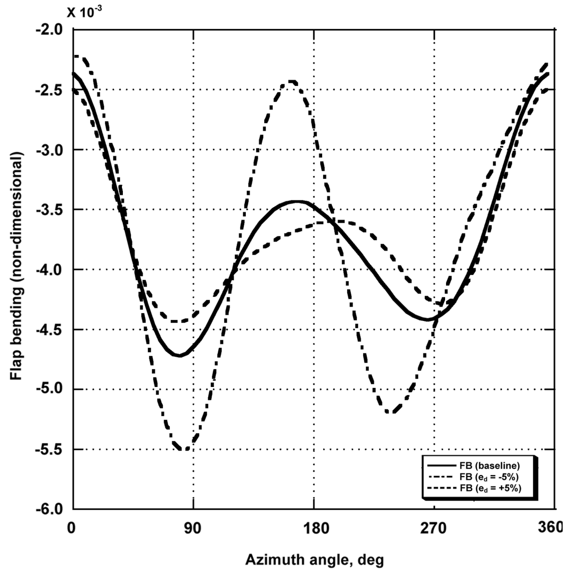


b) Lag displacement

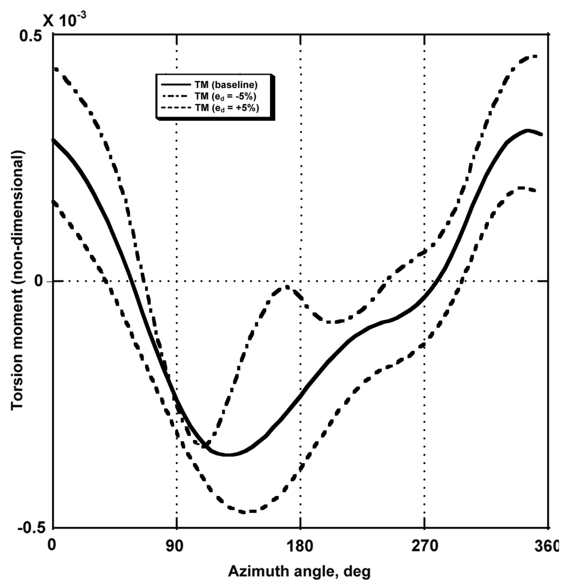


c) Elastic twist angles

Fig. 5 Stochastic distribution of tip responses of composite blades with the manufacturing uncertainty.



a) Flap bending (FB) moment



b) Torsion moment (TM)

Fig. 6 Effect of elastic-axis offsets on blade root moments.

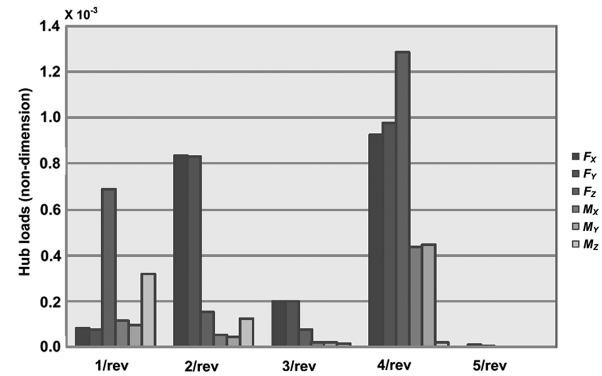


Fig. 7 Nondimensional hub loads (mean) for composite blades with the material uncertainty.

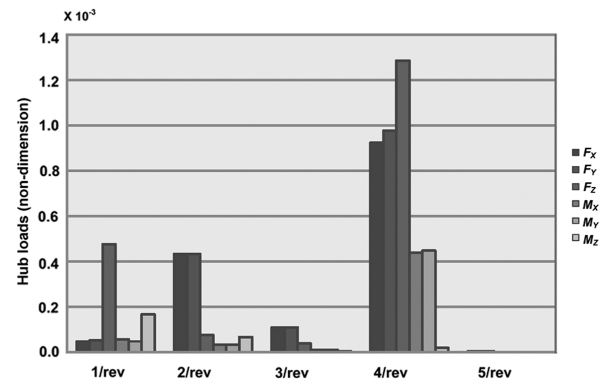


Fig. 8 Nondimensional hub loads (mean) for composite blades with the fabrication uncertainty.

in the stochastic analysis. The hub forces and moments are nondimensionalized by $m_0\Omega^2R^2$ and $m_0\Omega^2R^3$, respectively. As can be seen in Fig. 7, all the harmonics of hub loads as well as the 4/rev component appear due to the dissimilarity of the rotor system. This dissimilarity is induced by considering the stochastic material properties of composites. The major sources of the hub vibration are 4/rev; however, a similar order of magnitudes is obtained with the other harmonic components. The dominance of 1/rev and 2/rev forces of the rotor needs to be counteracted using the conventional track and balance technique [11] or the active vibration reduction methods [16,17]. The results shown in Fig. 7 are the case with the material uncertainty. Figure 8 shows the hub vibration results with the fabrication uncertainty. Even though the non-N_b/rev values for the fabrication uncertainty are fairly low as compared with those of the material uncertainty, they cannot be ignored. These results indicate that the influence of dissimilarity should be taken into account in the uncertainty qualification study for the aeroelastic analysis of composite rotor blades.

Figures 9 and 10 show the probability histograms obtained for each of the hub vibratory forces (Fig. 9) and moments (Fig. 10) with the material uncertainty case, denoted in terms of harmonic components up to 5/rev. To see the relative behavior of the histograms, the horizontal axis is expressed as a percentage deviation from the mean values. It is observed that the probability histograms exhibit a non-Gaussian-type of distribution, except the 4/rev hub load components, which show the normal-type distribution. The reason is because the unbalanced blade loads contribute to non-N_b/rev hub loads, resulting in a non-Gaussian-type distribution, whereas the balanced blade loads contribute to N_b/rev hub loads for a normal distribution. It is noted that the material properties and the resulting cross-sectional properties show the normal distribution. Furthermore, it is observed that the material uncertainty has only a marginal effect on the 4/rev harmonics, whereas the other harmonics become influenced significantly by the material uncertainty. The 4/rev harmonics show a maximum deviation of −34.8 to 32.1% (generally within ±4%), while most of other harmonics show a wide scatter (−99.1 to 408%) with apparent skewness to the negative side, which indicates smaller magnitudes than their mean values.

As shown above, the uncertainties in the cross-sectional properties of the rotor blades induce the dissimilarity in the rotor system, which brings extra harmonics of hub loads. Stochastic behaviors of the hub vibratory loads taking the material uncertainty into account are summarized in Table 3, in which the maximum values at 95% probability are presented for important harmonics influencing the vibration. Each hub load component is normalized with respect to the 4/rev hub loads (not shown). From the hub shear results, it is observed that 1/rev F_Z and 2/rev F_X and F_Y are the most affected shear forces for which the maximum values at 95% probability are about 1.05, 2.07, and 1.96 times higher than their respective 4/rev loads. In the case of hub vibratory moments, maximum variations are observed in all the harmonic components of M_Z. Even though the baseline values of 4/rev M_Z are quite low, which are about 22 times smaller than 4/rev M_X and M_Y, 1/rev M_Z shows a large variation that is about 1.4 times that of 4/rev M_X and M_Y. These results again indicate the importance of considering the non-N_b/rev harmonics in the hub vibration behavior of composite rotor blades when the uncertainties affect the rotor system.

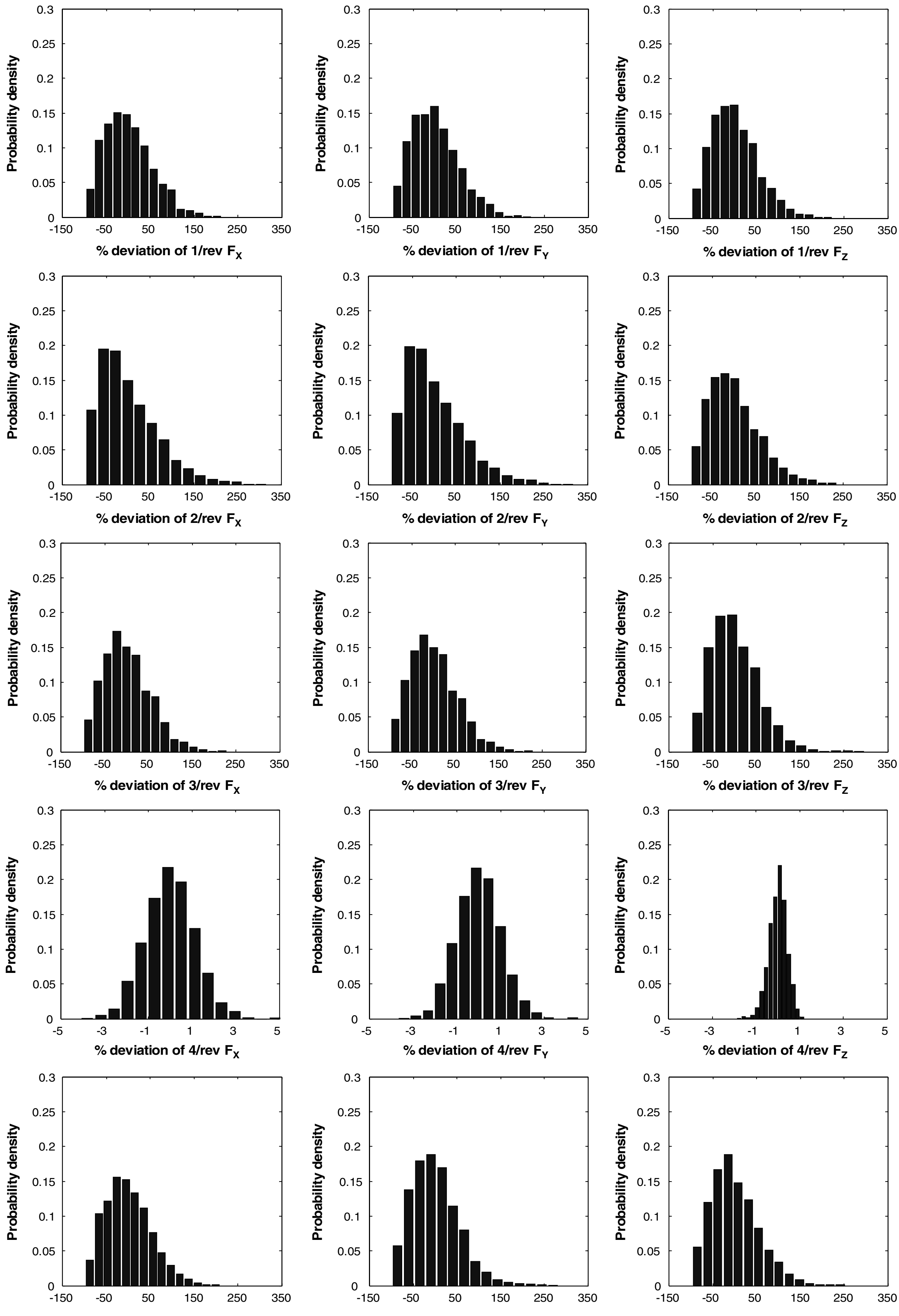


Fig. 9 Probability density of hub shear forces for composite blades with the material uncertainty.

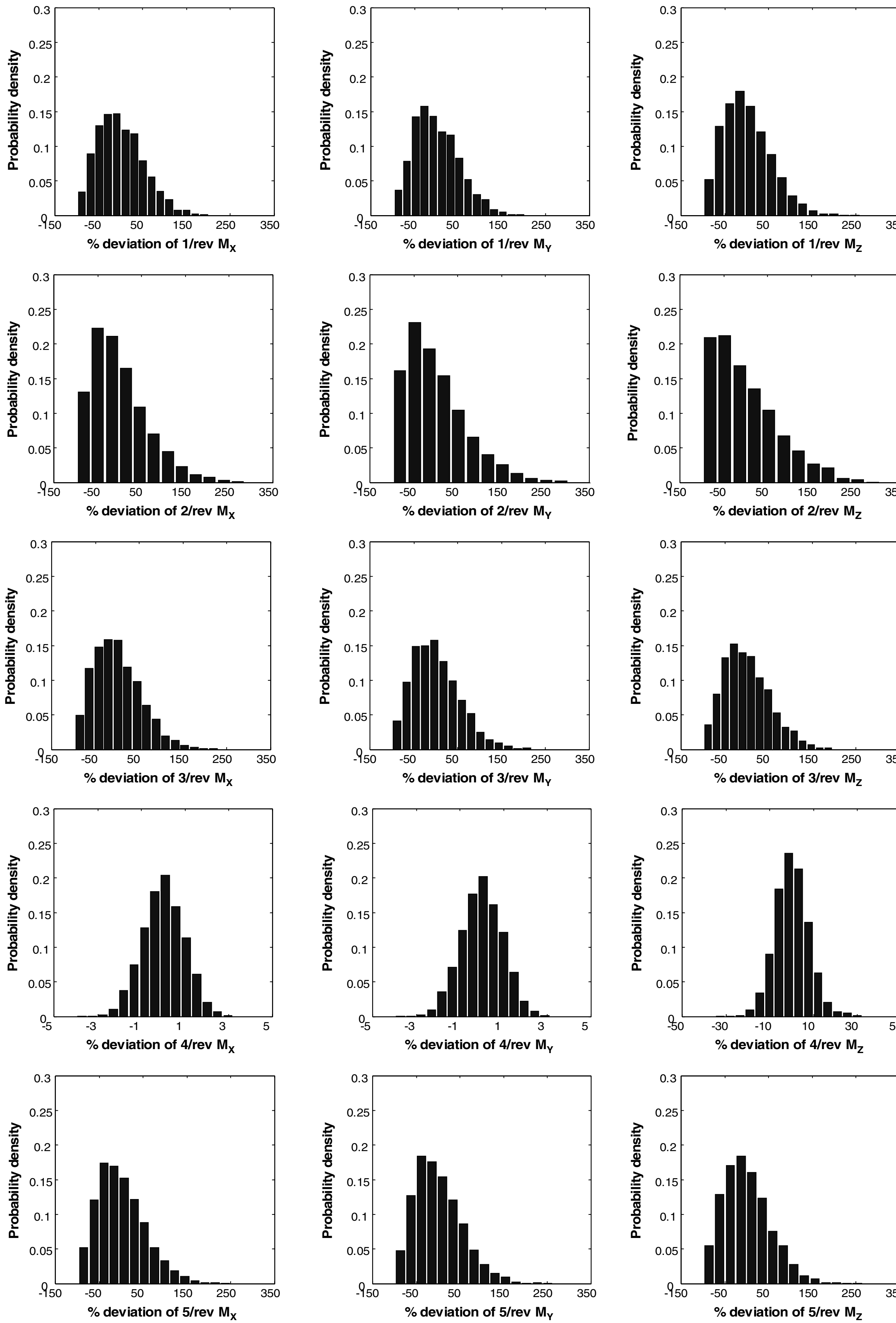


Fig. 10 Probability density of hub moments for composite blades with the material uncertainty.

Table 3 Maximum values at 95% probability for stochastic behavior of hub forces and moments

Harmonics	F_X [a]	F_Y [a]	F_Z [a]	M_X [b]	M_Y [b]	M_Z [b]
1/rev	0.172	0.151	1.055	0.523	0.410	31.01
2/rev	2.070	1.963	0.248	0.271	0.240	15.86
3/rev	0.417	0.359	0.118	0.097	0.097	1.343
5/rev	0.016	0.014	0.0031	0.0034	0.0032	0.050

[a]Nondimensionalized by $m_0\Omega^2R^2$
[b]Nondimensionalized by $m_0\Omega^2R^3$

IV. Conclusions

In this work, the stochastic behaviors of aeroelastic response and hub vibratory loads due to uncertainties in composite material properties, lamination geometries, and manufacturing stage of rotor blades are studied using the Monte Carlo simulation along with the comprehensive aeroelastic analysis system. The following conclusions are drawn from this investigation.

1) The material properties of composite laminas showing stochastic behavior in the form of normal distribution lead to the same form of stochastic behavior with different coefficient of variation of the cross-sectional stiffness properties.

2) The uncertainties in the cross-sectional properties induce the dissimilarity in the rotor system, which brings extra non-N_b/rev harmonics of vibratory loads. Even though the cross-sectional stiffness properties exhibit a normal distribution, the resulting histograms and probability distribution of non-N_b/rev hub loads show non-Gaussian-type distributions, due to the unbalanced components of blade loads in the dissimilar rotor system, whereas the N_b/rev hub loads exhibit the normal-type distribution due to the balanced components of blade loads.

3) The manufacturing uncertainty introduced by taking the elastic-axis offsets as random variables largely affects the aeroelastic response, especially for flap and torsion deformation. It is indicated that the drastic changes of aeroelastic responses of blades have no relation with the aeroelastic instability, but with the increase of aerodynamic loads, due to a greater offset value between the elastic axis and the aerodynamic center. It is found that the overall behavior of the blade loads exhibit a similar wave form as compared with the resulting responses.

4) Stochastic behaviors of the hub vibratory loads show that 1/rev F_Z, 2/rev F_X and F_Y, and all the harmonics in M_Z are the most affected vibratory harmonics. The maximum values of the shear forces at 95% probability are about 1.05, 2.07, and 1.96 times those of the respective 4/rev loads, whereas in case of hub moments, as much as 31-times-larger values of 1/rev M_Z in comparison with those of 4/rev are obtained.

Acknowledgments

This work was supported by National Research Foundation of Korea Grant funded by the Korean Government (2009-0074059). This work was also supported by National Research Foundation of Korea Grant funded by the Korean Government (K20601000001). The authors would like to express sincere thanks to the reviewers for their helpful and constructive comments.

References

[1] Jung, S. N., Nagaraj, V. T., and Chopra, I., "Assessment of Composite Rotor Blade Modeling Techniques," *Journal of the American Helicopter Society*, Vol. 44, No. 3, 1999, pp. 188–205. doi:10.4050/JAHS.44.188

[2] Hodges, D. H., "Review of Composite Rotor Blade Modeling," *AIAA Journal*, Vol. 28, No. 3, 1990, pp. 561–565. doi:10.2514/3.10430

[3] Friedmann, P. P., "Rotary-Wing Aeroelasticity: Current Status and Future Trends," *AIAA Journal*, Vol. 42, No. 10, 2004, pp. 1953–1972. doi:10.2514/1.9022

[4] Ganguli, R., "A Survey of Recent Developments in Rotorcraft Design Optimization," *Journal of Aircraft*, Vol. 41, No. 3, 2004, pp. 493–510. doi:10.2514/1.58

[5] Pettit, C. L., "Uncertainty Quantification in Aeroelasticity: Recent Results and Research Challenges," *Journal of Aircraft*, Vol. 41, No. 5, 2004, pp. 1217–1229. doi:10.2514/1.3961

[6] Kim, T. K., and Hwang, I. H., "Reliability Analysis of Composite Wing Subjected to Gust Loads," *Composite Structures*, Vol. 66, Nos. 1–4, 2004, pp. 527–531. doi:10.1016/j.compstruct.2004.04.072

[7] Pradlwarter, H. J., Pellissetti, M. F., Schenk, C. A., Schuller, G. I., Kreis, A., Fransen, S., Calvi, A., and Klein, M., "Realistic and Efficient Reliability Estimation for Aerospace Structures," *Computer Methods in Applied Mechanics and Engineering*, Vol. 194, Nos. 12–16, 2005, pp. 1597–1617. doi:10.1016/j.cma.2004.05.029

[8] Koutsourelakis, P. S., Kuntiyawichai, K., and Schuller, G. I., "Effect of Material Uncertainties on Fatigue Life Calculations of Aircraft Fuselages: A Cohesive Element Model," *Engineering Fracture Mechanics*, Vol. 73, No. 9, 2006, pp. 1202–1219. doi:10.1016/j.engfracmech.2006.01.003

[9] Murugan, S., Ganguli, R., and Harursampath, D., "Aeroelastic Response of Composite Helicopter Rotor with Random Material Properties," *Journal of Aircraft*, Vol. 45, No. 1, 2008, pp. 306–322. doi:10.2514/1.30180

[10] Murugan, S., Harursampath, D., and Ganguli, R., "Material Uncertainty Propagation in Helicopter Nonlinear Aeroelastic Response and Vibration Analysis," *AIAA Journal*, Vol. 46, No. 9, 2008, pp. 2332–2344. doi:10.2514/1.35941

[11] Rosen, A., and Ben-Ari, R., "Mathematical Modelling of a Helicopter Rotor Track and Balance: Theory," *Journal of Sound and Vibration*, Vol. 200, 1997, pp. 589–603. doi:10.1006/jsvi.1996.0669

[12] Wang, J. M., and Chopra, I., "Dynamics of Helicopters with Dissimilar Blades in Forward Flight," 17th European Rotorcraft Forum, Berlin, Germany, Paper ERF91-12, 24–27 Sept. 1991.

[13] Ganguli, R., Chopra, I., and Haas, D. J., "Simulation of Helicopter Rotor-System Damage, Blade Mistracking, Friction, and Freeplay," *Journal of Aircraft*, Vol. 35, 1998, pp. 591–597. doi:10.2514/2.2342

[14] Yang, M., Chopra, I., and Haas, D. J., "Vibration Prediction for Rotor System with Faults Using Coupled Rotor-Fuselage Model," *Journal of Aircraft*, Vol. 41, 2004, pp. 348–358. doi:10.2514/1.9330

[15] Singh, H. K., Pawar, P. M., Ganguli, R., and Jung, S. N., "On the Effect of Mass and Stiffness Unbalance on Helicopter Tail Rotor System Behavior," *Aircraft Engineering and Aerospace Technology*, Vol. 80, No. 2, 2008, pp. 129–138. doi:10.1108/00022660810859355

[16] Roget, B., and Chopra, I., "Individual Blade Control Methodology for a Rotor with Dissimilar Blades," *Journal of the American Helicopter Society*, Vol. 48, No. 3, 2003, pp. 176–185. doi:10.4050/JAHS.48.176

[17] Pawar, P. M., and Jung, S. N., "Active Twist Control Methodology for Vibration Reduction of a Helicopter with Dissimilar Rotor System," *Smart Materials and Structures*, Vol. 18, 2009, Paper 035013. doi:10.1088/0964-1726/18/3/035013

[18] Jung, S. N., Nagaraj, V. T., and Chopra, I., "Refined Structural Model for Thin- and Thick-Walled Composite Rotor Blades," *AIAA Journal*, Vol. 40, No. 1, 2002, pp. 105–116. doi:10.2514/2.1619

[19] Jung, S. N., Park, I. J., and Shin, E. S., "Theory of Thin-Walled Composite Beams with Single and Double-Cell Sections," *Composites. Part B, Engineering*, Vol. 38, 2007, pp. 182–192. doi:10.1016/j.compositesb.2006.07.001

[20] Bir, G., and Chopra, I., "University of Maryland Advanced Rotorcraft Code (UMARC): Theory Manual," Univ. of Maryland, Rept. 94-18, College Park, MD, 1994.

[21] Leishman, J. G., and Beddoes, T. S., "A Semi-Empirical Model for Dynamic Stall," *Journal of the American Helicopter Society*, Vol. 34, 1989, pp. 3–17. doi:10.4050/JAHS.34.3

[22] Bagai, A., and Leishman, J. G., "Rotor Free-Wake Modeling Using a Pseudo-Implicit Technique—Including Comparisons with Experimental Data," *Journal of the American Helicopter Society*, Vol. 40, No. 3, 1995, pp. 29–41. doi:http://dx.doi.org/10.4050/JAHS.40.29

[23] Jung, S. N., Kim, K. N., and Kim, S. J., "Forward Flight Stability Characteristics for Composite Hingeless Rotors with Transverse Shear Deformation," *AIAA Journal*, Vol. 40, No. 9, 2002, pp. 1717–1725. doi:10.2514/2.1875

[24] Smith, E. C., "Vibration and Flutter of Stiff In-Plane Elastically Tailored Composite Rotor Blades," *Mathematical and Computer Modelling*, Vol. 19, Nos. 3–4, 1994, pp. 27–45. doi:10.1016/0895-7177(94)90055-8

[25] Antonio, C. C., and Hoffbauer, L. N., "From Local to Global Importance Measures of Uncertainty Propagation in Composite Structures," *Composite Structures*, Vol. 85, 2008, pp. 213–225. doi:10.1016/j.compstruct.2007.10.012

[26] Shaker, A., Abdelrahman, W. G., Tawfik, A., and Sadek, E., "Stochastic Finite Element Analysis of the Free Vibration of Laminated Composite Plates," *Computational Mechanics*, Vol. 41, No. 4, 2008, pp. 493–501. doi:10.1007/s00466-007-0205-7

[27] Onkar, A. K., and Yadav, D., "Forced Nonlinear Vibration of Laminated Composite Plates with Random Material Properties," *Composite Structures*, Vol. 70, No. 3, 2005, pp. 334–342. doi:10.1016/j.compstruct.2004.08.037

[28] Vinckenroy, G. V., and de Wilde, W. P., "The Use of Monte Carlo Techniques in Statistical Finite Element Methods for the Determination of the Structural Behaviour of Composite Materials Structural Components," *Composite Structures*, Vol. 32, Nos. 1–4, 1995, pp. 247–253. doi:10.1016/0263-8223(95)00055-0